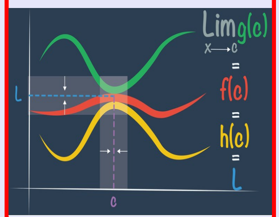
Math 261

Spring 2023

Lecture 18



Feb 19-8:47 AM

First Derivative of $f(x)$:

$$f'(x), \quad \frac{d}{dx}[f(x)]$$

Second Derivative of $f(x)$:

$$f''(x), \quad \frac{d^2}{dx^2}[f(x)] = \frac{d}{dx}[f'(x)]$$

Suppose $f(x) = x^2 - 8x + 6$

$$f'(x) = 2x - 8 + 0 = \boxed{2x - 8}$$

$$f''(x) = \frac{d}{dx}[f'(x)] = \frac{d}{dx}[2x - 8] = 2 - 0 = \boxed{2}$$

Mar 8-8:46 AM

Suppose $f(x) = \underbrace{(2x-5)(5x+2)}_{\text{Product}}$

$$f'(x) = \underbrace{\frac{d}{dx}[2x-5]\cdot(5x+2) + (2x-5)\cdot\frac{d}{dx}[5x+2]}_{\text{Product Rule}}$$

$$= 2(5x+2) + (2x-5)\cdot 5$$

$$= 10x + 4 + 10x - 25 = \boxed{20x - 21}$$

$$f''(x) = \frac{d}{dx}[f'(x)] = \frac{d}{dx}[20x-21] = \boxed{20}$$

Mar 8-8:50 AM

Given $f(x) = x^2 \sin x$

$$f'(x) = \frac{d}{dx}[x^2]\cdot \sin x + x^2\cdot\frac{d}{dx}[\sin x]$$

$$= \boxed{2x \sin x + x^2 \cdot \cos x}$$

$$f''(x) = \frac{d}{dx}[2x \sin x] + \frac{d}{dx}[x^2 \cos x]$$

$$= 2[1\cdot \sin x + x\cdot \cos x] + 2x\cdot \cos x + x^2\cdot(-\sin x)$$

$$= 2\sin x + 2x\cos x + 2x\cos x - x^2 \sin x$$

$$= \boxed{2\sin x + 4x\cos x - x^2 \sin x}$$

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$$f(x) = \frac{\sin x}{1 + \cos x}$$

$$f'(x) = \frac{\frac{d}{dx}[\sin x] \cdot (1 + \cos x) - \sin x \cdot \frac{d}{dx}[1 + \cos x]}{(1 + \cos x)^2}$$

$$= \frac{\cos x \cdot (1 + \cos x) - \sin x \cdot (-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2} = \frac{1 + \cos x}{(1 + \cos x)^2}$$

$$\boxed{f'(x) = \frac{1}{1 + \cos x}}$$

Now $f''(x)$

$$f''(x) = \frac{\cancel{\frac{d}{dx}[1]} \cdot (1 + \cos x) - 1 \cdot \frac{d}{dx}[1 + \cos x]}{(1 + \cos x)^2}$$

$$= \frac{-1(-\sin x)}{(1 + \cos x)^2}$$

$$\boxed{f''(x) = \frac{\sin x}{(1 + \cos x)^2}}$$

Mar 8-8:58 AM

$$f(x) = \frac{x}{x - 2}$$

1) Domain $(-\infty, 2) \cup (2, \infty)$

$x - 2 \neq 0$

$x \neq 2$

2) $f(0) = \frac{0}{0 - 2} = \frac{0}{-2} = \boxed{0}$

3) $$f'(x) = \frac{\frac{d}{dx}[x] \cdot (x - 2) - x \cdot \frac{d}{dx}[x - 2]}{(x - 2)^2}$$

$$= \frac{x - 2 - x}{(x - 2)^2} = \frac{-2}{(x - 2)^2}$$

$$f'(x) = \frac{-2}{x^2 - 4x + 4}$$

4) $f'(0)$

$$= \frac{-2}{(0 - 2)^2} = \frac{-2}{4}$$

$$= \boxed{\frac{-1}{2}}$$

5) Find $f''(x)$

$$f''(x) = \frac{\cancel{\frac{d}{dx}[-2]} \cdot (x^2 - 4x + 4) - (-2) \cdot \frac{d}{dx}[x^2 - 4x + 4]}{(x^2 - 4x + 4)^2}$$

$$[(x - 2)^2]^2$$

$$= \frac{2(2x - 4)}{(x - 2)^4} = \frac{2 \cdot 2\cancel{(x - 2)}}{(x - 2)^{\cancel{4}\,3}} = \frac{4}{(x - 2)^3}$$

6) $f''(0)$

$$f''(0) = \frac{4}{(0 - 2)^3} = \frac{4}{-8} = \boxed{\frac{-1}{2}}$$

Mar 8-9:05 AM

Find $f'(x)$

1) $f(x) = 10$ $\boxed{f'(x) = 0}$

Constant

2) $f(x) = 10t$ $\boxed{f'(x) = 0}$ $\frac{d}{dx}[f(x)]$

Variable x

3) $f(x) = 10tx$ $\boxed{f'(x) = 10t}$

4) $f(x) = \sin^2 x + \cos^2 x = 1$ $\boxed{f'(x) = 0}$

Mar 8-9:18 AM

Suppose $y(t) = \sqrt{t} = t^{1/2}$

$$\frac{d}{dx}[x^n] = n x^{n-1}$$

$$y'(t) = \frac{dy}{dt} = \frac{d}{dt}\left[t^{1/2}\right]$$

Power Rule

$$= \frac{1}{2} t^{1/2 - 1} = \frac{1}{2} t^{-1/2} = \frac{1}{2t^{1/2}}$$

$$= \frac{1}{2\sqrt{t}}$$

Mar 8-9:23 AM

$f(x) = \tan x + 2$

1) $f\left(\frac{\pi}{4}\right) = \boxed{\tan\frac{\pi}{4}} + 2 = 1 + 2 = \boxed{3}$

$\tan\frac{\pi}{4} \to 1$

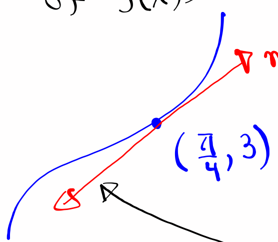
$f(x) = \tan x + 2$

2) $f'\left(\frac{\pi}{4}\right)$

$f'(x) = \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = \left[\sec\frac{\pi}{4}\right]^2 = (\sqrt{2})^2 = \boxed{2}$

3) Eqn of tan. line at $x = \frac{\pi}{4}$ to the graph of $f(x) = \tan x + 2$.



$m = f'\left(\frac{\pi}{4}\right) = 2$

$y - y_1 = m(x - x_1)$

$y - 3 = 2\left(x - \frac{\pi}{4}\right)$

$\boxed{y = 2x + 3 - \frac{\pi}{2}}$

Mar 8-9:26 AM

Class QZ 4:

1) Evaluate $\lim\limits_{x\to\infty} \dfrac{5 - 2x}{4x + 1} = \dfrac{-\infty}{\infty}$

$$\lim_{x\to\infty} \frac{5-2x}{4x+1} = \lim_{x\to\infty} \frac{\frac{5}{x} - \frac{2x}{x}}{\frac{4x}{x} + \frac{1}{x}} = \lim_{x\to\infty} \frac{\cancel{\frac{5}{x}}^{\to 0} - 2}{4 + \cancel{\frac{1}{x}}^{\to 0}} = \frac{-2}{4} = \boxed{\frac{-1}{2}}$$

2) Find $f'(x)$ for $f(x) = \dfrac{2x^2}{x^2 - 1}$

$$f'(x) = \frac{4x(x^2-1) - 2x^2(2x)}{(x^2-1)^2} = \frac{4x^3 - 4x - 4x^3}{(x^2-1)^2}$$

$$= \boxed{\frac{-4x}{(x^2-1)^2}}$$

Mar 8-9:35 AM